

TOWARDS DIFFERENTIAL TOP PAIR PRODUCTION AT NNLO

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Based on work done in collaboration with:

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Loop Fest XIII - NYCCT, New York, June 20, 2014

Top Pair Production At The LHC

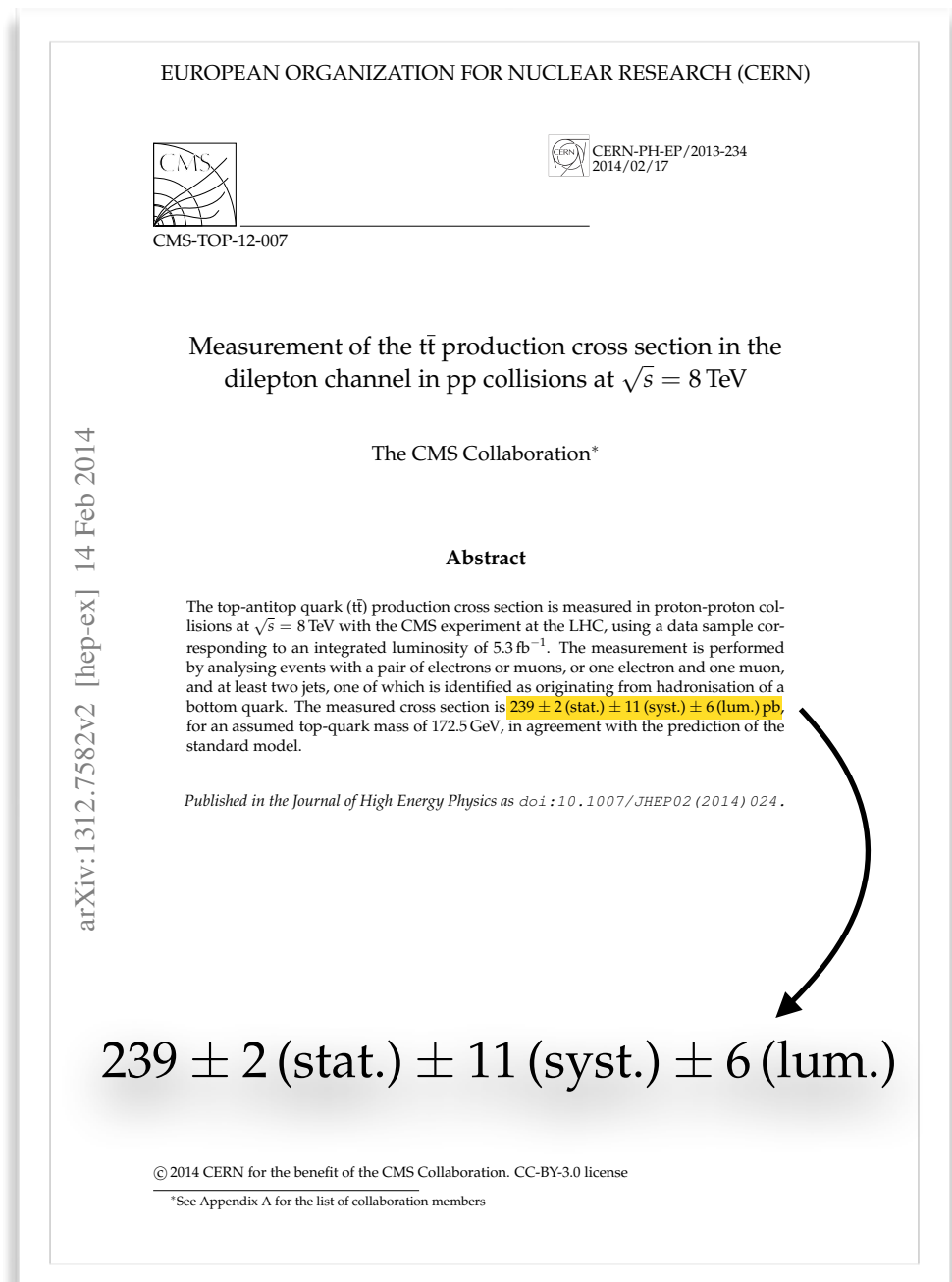
- Interesting signal. Rich phenomenology. Important in new physics searches...
- Top quark pairs are **copiously produced at the LHC**

$$\sigma_{t\bar{t}+X}(\sqrt{s} = 7 \text{ TeV}) \sim 170 \text{ pb}$$

$$\sigma_{t\bar{t}+X}(\sqrt{s} = 8 \text{ TeV}) \sim 250 \text{ pb}$$

$$\sigma_{t\bar{t}+X}(\sqrt{s} = 14 \text{ TeV}) \sim 950 \text{ pb}$$

- Abundant statistics. **Expected experimental error ~5%**
- Need **theoretical predictions** with similar accuracy
 - Requires computations through **higher orders in perturbation theory**



Top Pair Production At The LHC: State Of The Art

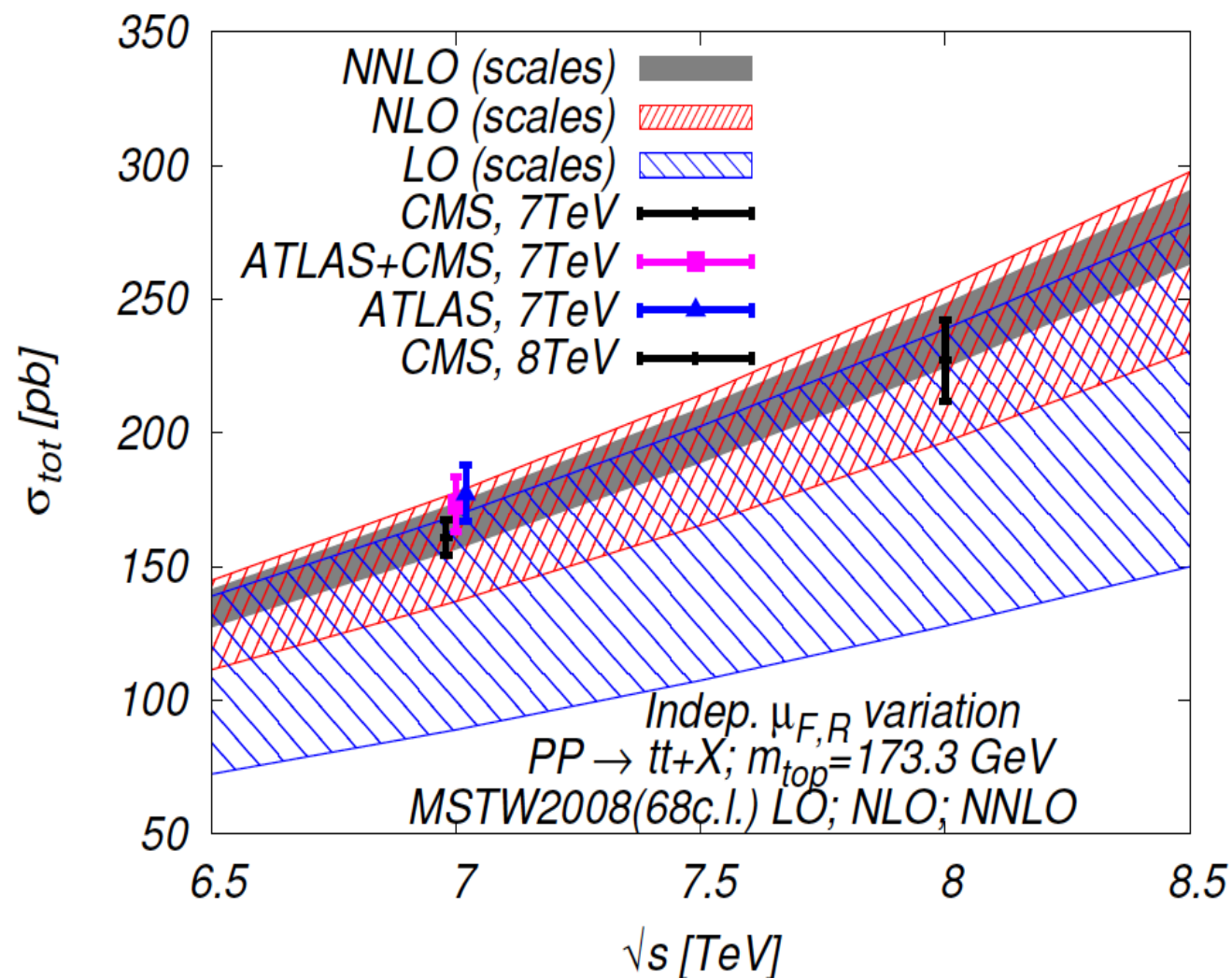
- NLO QCD corrections: Ellis, Dawson, Nason; Beenakker, Kuijf, van Neerven, Smith '89
- NLO EW corrections: Beenakker, Bernreuther, Denner, Fuecker, Hollik, Kao, Kollar, Kühn, Ladinsky, Mertig, Moretti, Nolten, Ross, Sack, Scharf, Si, Uwer, Wackenroth, Yuan
- Threshold resummation and Coulomb corrections: Ahrens, Banfi, Berger, Bonciani, Catani, Contopanagos, Czakon, Ferroglia, Frixione, Kidonakis, Kiyo, Kühn, Laenen, Mangano, Mitov, Moch, Nason, Neubert, Pecjak, Ridolfi, Steinhauser, Sterman, Uwer, Vogt, Yang

Yield a theoretical uncertainty of $\sim 10\%$

To match theory and experimental accuracies at the LHC, cross sections for top pair production must be calculated through NNLO in pQCD

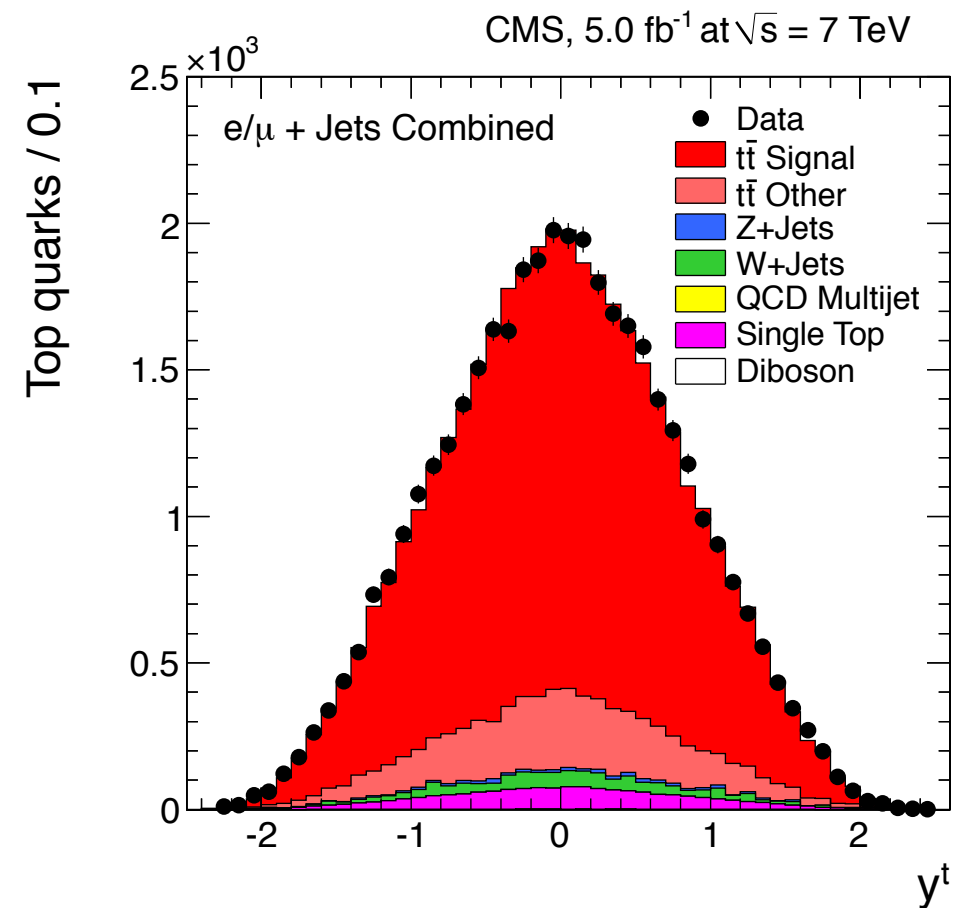
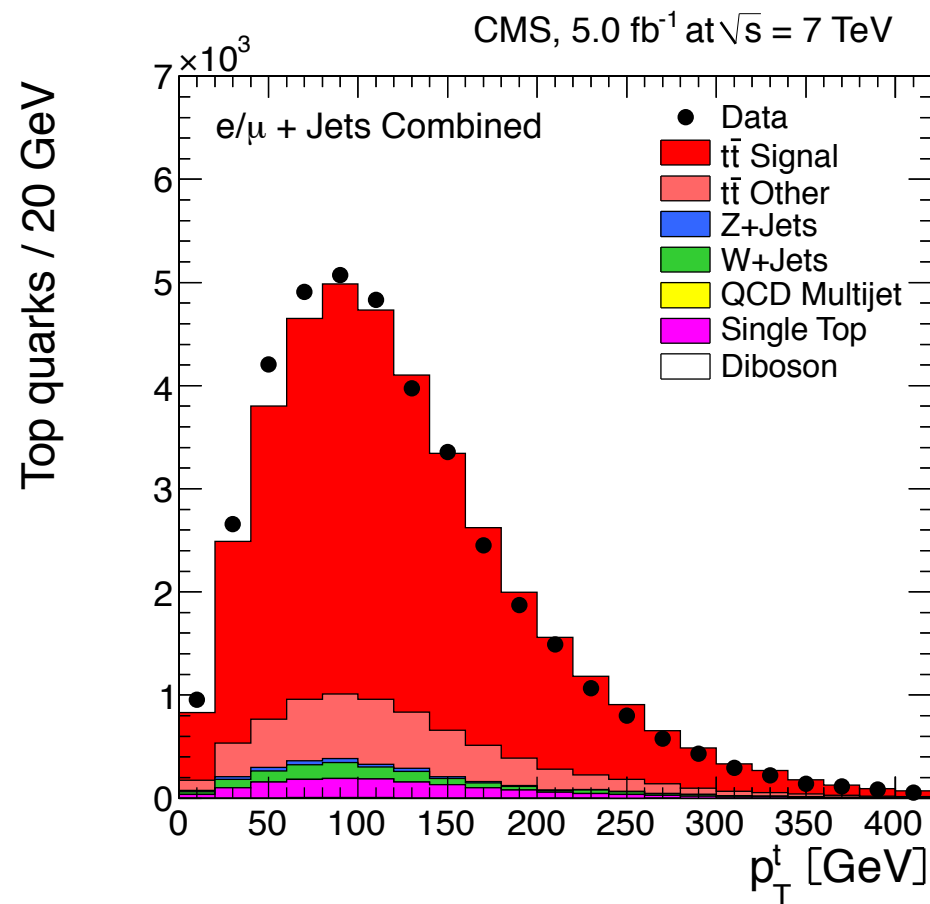
Top Pair Production At The LHC: State Of The Art

- Calculation of the **total NNLO cross section completed** [Czakon, Fiedler, Mitov '13]
 - Combined with NNLL resummation
 - Theoretical and experimental uncertainties of similar sizes (percent level)



Differential Top Pair Production

- Differential distributions **probe the dynamics of top quark production**
 - Important in order to **search for new physics** as deviations from SM predictions



Need NNLO predictions for $\frac{d\sigma}{dX}$ with $X = p_T^t, p_T^{t\bar{t}}, y^t, y^{t\bar{t}}, m_{t\bar{t}}$

- Approximate results available, including decays (A. Broggio's talk)

Differential Top Pair Production

- Goal: fully differential event generator for $t\bar{t}$ production at NNLO
- This talk:
 - Status of our NNLO calculation for the $q\bar{q}$ channel (leading-color + N_l only)

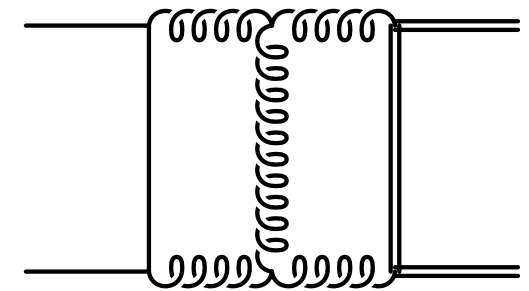
$$\begin{aligned} d\hat{\sigma}_{q\bar{q},NNLO} = N_c C_F \bigg[& N_c^2 A + B + \frac{C}{N_c^2} + N_l \left(N_c D_l + \frac{E_l}{N_c} \right) + N_h \left(N_c D_h + \frac{E_h}{N_c} \right) \\ & + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h \bigg] \end{aligned}$$

- Preliminary differential distributions as a proof of principle

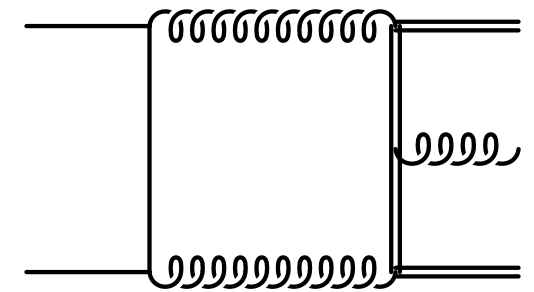
Ingredients For Top Pair Production At NNLO

- **LO and NLO** fully differential cross sections
 - ▶ Known [Ellis, Dawson, Nason '89; Beenakker, Kuijf, van Neerven, Smith '89]
 - ▶ Re-derived using NLO antenna subtraction [GA, Gehrmann-De Ridder '11]

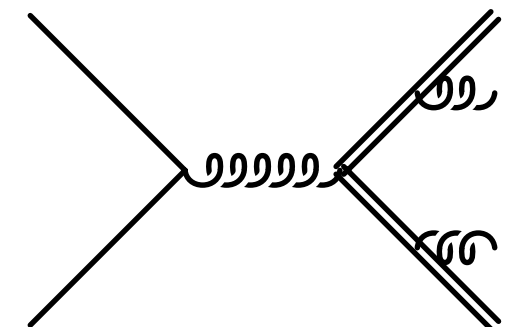
- **Two-loop** $2 \rightarrow 2$ matrix elements
 - ▶ Use **analytic results** (A. von Manteuffel's talk)
[Bonciani, Ferroglia, Gehrmann, Maître, v. Manteuffel, Studerus]



- **One-loop** $2 \rightarrow 3$ matrix elements
 - ▶ Obtained numerically with **OpenLoops**
[Cascioli, Meierhöfer, Pozzorini]
 - ✓ Color structure handled algebraically
 - ✓ **Quadruple precision** evaluation in soft limit



- **Tree-level** $2 \rightarrow 4$ matrix elements



Ingredients For Top Pair Production At NNLO

$$d\hat{\sigma}_{NNLO} = \int_{d\Phi_4} d\hat{\sigma}_{NNLO}^{RR} + \int_{d\Phi_3} \left(d\hat{\sigma}_{NNLO}^{RV} + d\hat{\sigma}_{NNLO}^{MF,1} \right) + \int_{d\Phi_2} \left(d\hat{\sigma}_{NNLO}^{VV} + d\hat{\sigma}_{NNLO}^{MF,2} \right)$$

- $d\hat{\sigma}_{NNLO}^{RV}$, $d\hat{\sigma}_{NNLO}^{VV}$ \longrightarrow **explicit IR poles** from loop integration
- $\int_{d\Phi_4} d\hat{\sigma}_{NNLO}^{RR}$, $\int_{d\Phi_3} d\hat{\sigma}_{NNLO}^{RV}$ \longrightarrow **implicit IR poles** from PS integration over single and double unresolved regions

Need a procedure to **isolate and cancel all IR singularities**, and assemble all parts in a **parton-level event generator**

Antenna Subtraction At NNLO

$$\begin{aligned} d\hat{\sigma}_{NNLO} = & \int_{d\Phi_4} \left(d\hat{\sigma}_{NNLO}^{RR} - d\hat{\sigma}_{NNLO}^S \right) \\ & + \int_{d\Phi_3} \left(d\hat{\sigma}_{NNLO}^{RV} - d\hat{\sigma}_{NNLO}^{VS} + d\hat{\sigma}_{NNLO}^{MF,1} + \int_1 d\hat{\sigma}_{NNLO}^{S,1} \right) \\ & + \int_{d\Phi_2} \left(d\hat{\sigma}_{NNLO}^{VV} + d\hat{\sigma}_{NNLO}^{MF,2} + \int_1 d\hat{\sigma}_{NNLO}^{VS} + \int_2 d\hat{\sigma}_{NNLO}^{S,2} \right) \end{aligned}$$

- Introduce **double real and real-virtual subtraction terms** $d\hat{\sigma}_{NNLO}^S$, $d\hat{\sigma}_{NNLO}^{VS}$ and add them back in integrated form
- The integrated double real subtraction term is split as

$$\int_{d\Phi_{m+2}} d\hat{\sigma}_{NNLO}^S = \int_{d\Phi_{m+1}} \int_1 d\hat{\sigma}_{NNLO}^{S,1} + \int_{d\Phi_m} \int_2 d\hat{\sigma}_{NNLO}^{S,2}$$

- Each PS integrand is **free of explicit poles, well behaved** in singular regions, and **can be integrated numerically in D=4**

Double Real Subtraction Terms

Use a **color decomposition** of the double real ($2 \rightarrow (m+2)$) amplitude

$$d\hat{\sigma}_{NNLO}^{RR} = \mathcal{N}_{NNLO}^{RR} \sum_{\text{perms}} d\Phi_{m+2}(p_3, \dots, p_{m+4}; p_1, p_2) |\mathcal{M}_{m+4}^0(\hat{1}, \hat{2}, 3, \dots)|^2 J_m^{(m+2)}(p_3, \dots, p_{m+4})$$

- $|\mathcal{M}_{m+4}^0|^2$ singular in **single and double unresolved limits** (soft, collinear, ...)
- ▶ **Factorization** in IR singular limits well known. Described by **universal unresolved factors** [Campbell, Glover '98; Catani Grazzini '99]. E.g. single and double soft limits:

$$|\mathcal{M}_{m+4}^0(\dots, p_i, p_j, p_k, \dots)|^2 \xrightarrow{p_j \rightarrow 0} \mathcal{S}(i, j, k) |\mathcal{M}_{m+3}^0(\dots, p_i, p_k, \dots)|^2$$

$$|\mathcal{M}_{m+4}^0(\dots, p_i, p_j, p_k, p_l, \dots)|^2 \xrightarrow{p_j, p_k \rightarrow 0} \mathcal{S}(i, j, k, l) |\mathcal{M}_{m+2}^0(\dots, p_i, p_l, \dots)|^2$$

- Unresolved factors captured by three and four-parton **tree-level antenna functions**

$$X_3^0(i, j, k) = S_{ijk, IK} \frac{|\mathcal{M}_3^0(i, j, k)|^2}{|\mathcal{M}_2^0(I, K)|^2}$$

One unresolved parton (j)

$$X_4^0(i, j, k, l) = S_{ijkl, IL} \frac{|\mathcal{M}_4^0(i, j, k, l)|^2}{|\mathcal{M}_2^0(I, L)|^2}$$

Two unresolved partons (j,k)

Double Real Subtraction Terms

- $d\hat{\sigma}_{NNLO}^S$ contains **three different antenna structures**

- ▶ $X_3^0(i, j, k) |\mathcal{M}_{m+3}(\dots, p_I, p_K, \dots)|^2 \longrightarrow$ Single unresolved limits
- ▶ $X_4^0(i, j, k, l) |\mathcal{M}_{m+2}(\dots, p_I, p_L, \dots)|^2 \longrightarrow$ Color-connected double unresolved limits
- ▶ $X_3^0 X_3'^0 |\mathcal{M}_{m+2}^0|^2 \longrightarrow$ (Almost) Color-unconnected double unresolved limits

- $3 \rightarrow 2$ and $4 \rightarrow 2$ **on-shell momentum mappings**

$$\{p_i, p_j, p_k\} \rightarrow \{p_I, p_K\} \quad \{p_i, p_j, p_k, p_l\} \rightarrow \{p_I, p_L\}$$

- ▶ **Conserve momentum** in reduce matrix elements
- ▶ Collapse to **appropriate kinematics in each unresolved limit**

Real-Virtual Subtraction Terms

Use a **color decomposition** of the tree and loop ($2 \rightarrow (m+1)$) amplitudes

$$d\hat{\sigma}_{NNLO}^{RV} = \mathcal{N}_{NNLO}^{RV} \sum_{\text{perms}} d\Phi_{m+1}(p_3, \dots, p_{m+3}; p_1, p_2) |\mathcal{M}_{m+3}^1(\hat{1}, \hat{2}, 3, \dots)|^2 J_m^{(m+1)}(p_3, \dots, p_{m+3})$$

- $|\mathcal{M}_{m+3}^1|^2$ singular in **single unresolved limits**. Well known factorization [Bern, Catani, Dixon, Dunbar, Kosower, Uwer, ...]

$$|\mathcal{M}_{m+3}^1|^2 \xrightarrow{j \text{ unresolved}} \text{Sing}^0 \times |\mathcal{M}_{m+2}^1|^2 + \text{Sing}^1 \times |\mathcal{M}_{m+2}^0|^2$$

- Accordingly, $d\hat{\sigma}_{NNLO}^{VS}$ is constructed as

$$|\mathcal{M}_{m+3}^1(\dots, p_i, p_j, p_k, \dots)|^2 \rightarrow X_3^0(i, j, k) |\mathcal{M}_{m+2}^1(\dots, p_I, p_K, \dots)|^2 + X_3^1(i, j, k) |\mathcal{M}_{m+2}^0(\dots, p_I, p_K, \dots)|^2$$

- One-loop antennae

$$X_3^1(i, j, k) = S_{ijk, IK} \frac{|\mathcal{M}_3^1(i, j, k)|^2}{|\mathcal{M}_2^0(I, K)|^2} - X_3^0(i, j, k) \frac{|\mathcal{M}_2^1(I, K)|^2}{|\mathcal{M}_2^0(I, K)|^2}$$

Integrated Subtraction Terms

Subtraction terms must be integrated and added back in lower multiplicity final states

- Phase space factorization (different in f-f, i-f, and i-i cases). E.g.

$$d\Phi_{m+2}(\dots, p_i, p_j, p_k, p_l, \dots) = d\Phi_m(\dots, p_I, p_L, \dots) \times d\Phi_{X_{ijkl}}(p_i, p_j, p_k, p_l)$$

- Integrated antennae are the inclusive integrals

$$\mathcal{X}_{ijkl}^0(\epsilon, s_{IL}) = \frac{1}{C(\epsilon)^2} \int d\Phi_{X_{ijkl}}(p_i, p_j, p_k, p_l) X_4^0(i, j, k, l)$$

- Integrated subtraction terms given by

$$\int_2 d\hat{\sigma}_{NNLO}^{S,2} \sim C(\epsilon)^2 d\Phi_m(\dots, p_I, p_L, \dots) \mathcal{X}_{ijkl}^0(\epsilon, s_{IL}) |\mathcal{M}_{m+2}(\dots, p_I, p_L, \dots)|^2 J_m^{(m)}(\dots, p_I, p_L, \dots)$$

- $\int_1 d\hat{\sigma}_{NNLO}^{S,1}$ and $\int_1 d\hat{\sigma}_{NNLO}^{VS}$ obtained analogously

NNLO Antenna Subtraction With Massive Quarks

Slide from J. Currie's talk

NNLO dijets at the LHC
└ Antenna Subtraction

Antenna Subtraction Toolbox

Many tools needed for implementation:

- ▶ final-final phase space mappings [Kosower '03]
 - ▶ FF X_3^0, X_4^0, X_3^1 antennae [Gehrmann-De Ridder, Gehrmann, Glover, '04, '05]
 - ▶ integrated FF antennae [Gehrmann-De Ridder, Gehrmann, Glover, '05]
- $\Rightarrow e^+e^- \rightarrow 3 \text{ jets at NNLO}$ [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, '07, Weinzierl '08]

Since then, extended for hadronic initial-states:

- ▶ initial-final + initial-initial mappings [Daleo, Gehrmann, Maître, '07]
- ▶ integrated IF X_3^1, X_4^0 [Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, '10]
- ▶ integrated II X_4^0 [Boughezal, Gehrmann-De Ridder, Ritzmann, '11. Gehrmann, Ritzmann, '12]
- ▶ integrated II X_3^1 [Gehrmann, Monni, '11]

All tools exist for hadron-hadron scattering

[Glover, Pires, '10. Gehrmann De-Ridder, Glover, Pires, '12. Gehrmann De-Ridder, Gehrmann, Glover, Pires, '13. JC, Glover, Wells, '13. JC, Gehrmann De-Ridder, Glover, Pires, '14.]

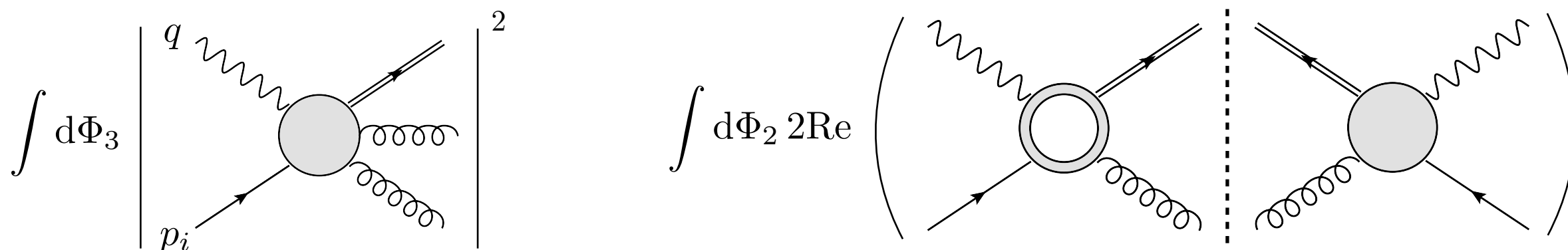
True for observables with
massless partons

Challenge: extend NNLO antenna subtraction method to treat massive quarks.

- Re-derive phase space mappings and factorizations
- Compute and integrate NLO and NNLO massive antennae

Integrated Massive Antennae

- Massless antennae: all known [Boughezal, Daleo, Gehrmann, Gehrmann-De Ridder, Glover, Luisoni, Maître, Ritzmann]
- Massive antennae: incomplete
 - ▶ Three parton tree-level: all known [Gehrmann-De Ridder, Ritzmann '09; GA, Gehrmann-De Ridder '11]
 - ▶ Four parton tree-level
 - ▶ Final-final: $\mathcal{A}_{Qgg\bar{Q}}^0, \mathcal{B}_{Qq\bar{q}\bar{Q}}^0$ [Bernreuther, Bogner, Dekkers]
 - ▶ Initial-final: $\mathcal{B}_{q,Qq'\bar{q}'}^0, \mathcal{E}_{g,Qq\bar{q}}^0, \tilde{\mathcal{E}}_{g,Qq\bar{q}}^0$ [GA, Dekkers, Gehrmann-De Ridder '12]
 - ▶ More integrated antennae needed for $q\bar{q} \rightarrow t\bar{t} + X$ at NNLO: $\mathcal{A}_{q,Qgg}^0, \mathcal{A}_{q,Qg}^1$



- ▶ Multiple scales: $m_{top}^2, q^2, p_i \cdot q$
- ▶ Coupled differential equations for master integrals

Subtraction Terms For Top Pair Production In The Quark-Antiquark Channel

Where we now stand:

	Leading color	N _l
$\int_{d\Phi_4} \left(d\hat{\sigma}_{NNLO}^R - d\hat{\sigma}_{NNLO}^S \right)$	✓ [1]	✓ [2]
$\int_{d\Phi_3} \left(d\hat{\sigma}_{NNLO}^{RV} - d\hat{\sigma}_{NNLO}^T \right)$	✓ [1]	✓ [3]
$\int_{d\Phi_2} \left(d\hat{\sigma}_{NNLO}^{VV} - d\hat{\sigma}_{NNLO}^U \right)$	✗*	✓ [3]

¹ [GA, Gehrmann-De Ridder, Meierhöfer, Pozzorini '14]

² [GA, Gehrmann-De Ridder '11]

³ [GA, Gehrmann-De Ridder (in preparation)]

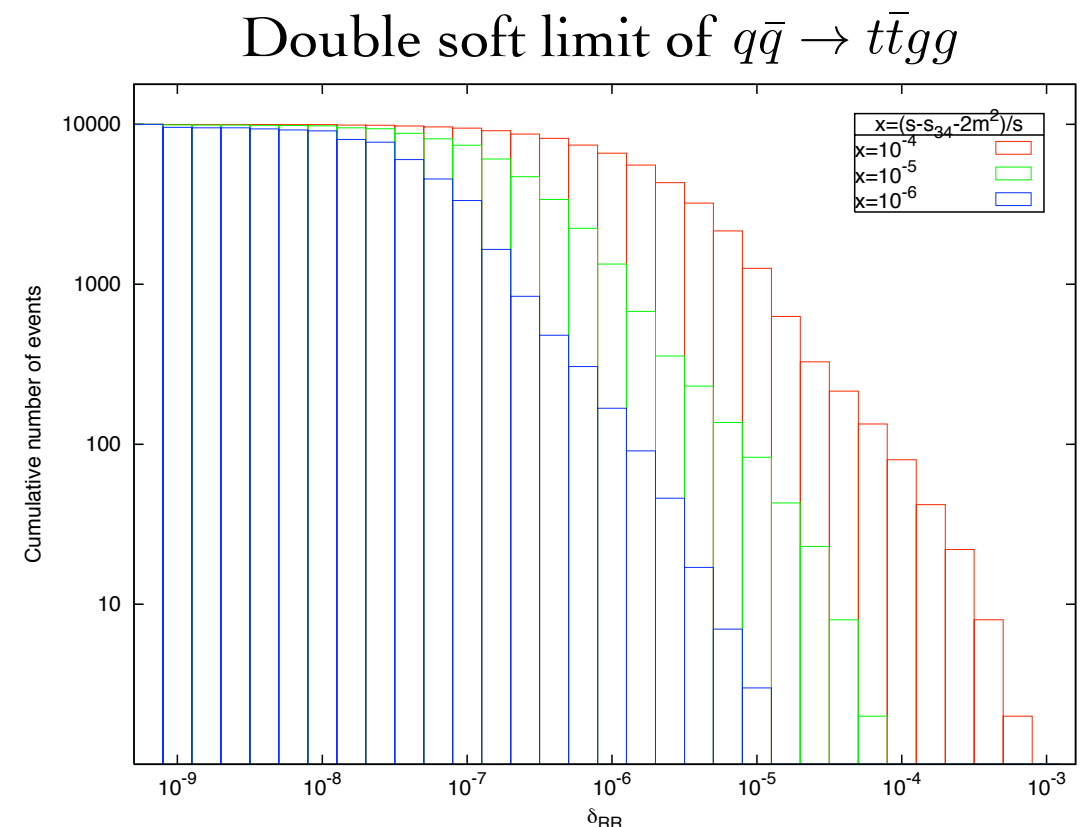
* Integrated massive antennae $\mathcal{A}_{q,Qgg}^0$ and $\mathcal{A}_{q,Qg}^1$ still missing. In progress.

Double Real Contributions

- Subtraction terms for partonic processes
 - ▶ $q\bar{q} \rightarrow t\bar{t}q'\bar{q}'$ [GA, Gehrmann-De Ridder '11]
 - ▶ $q\bar{q} \rightarrow t\bar{t}gg$ (leading-color only) [GA, Gehrmann-De Ridder, Meierhöfer, Pozzorini '14]
- Check of **convergence**
 - ▶ Generate events near every singular region
 - ▶ Control proximity to singularities with a control variable x (specific to each limit)
 - ▶ For each event, compute

$$\delta_{RR} = \left| \frac{d\hat{\sigma}_{NNLO}^{RR}}{d\hat{\sigma}_{NNLO}^S} - 1 \right|$$

- ▶ Convergence of $d\hat{\sigma}_{NNLO}^S$ to $d\hat{\sigma}_{NNLO}^{RR}$ observed in cumulative histograms in δ_{RR}
- Similar (good) **convergence observed in all single and double unresolved limits**



Real Virtual Contributions

- Partonic process $q\bar{q} \rightarrow t\bar{t}g$ at one-loop
- One-loop amplitudes computed
 - Numerically with **OpenLoops** for **leading-color** contributions
 - **Analytically** for **N_l** pieces
- Subtraction terms constructed and implemented
 - Leading-color: [GA, Gehrmann-De Ridder, Meierhöfer, Pozzorini '14]
 - N_l: [GA, Gehrmann-De Ridder (in preparation)]
- **Pointwise cancellation of explicit IR poles** checked **analytically** in both cases

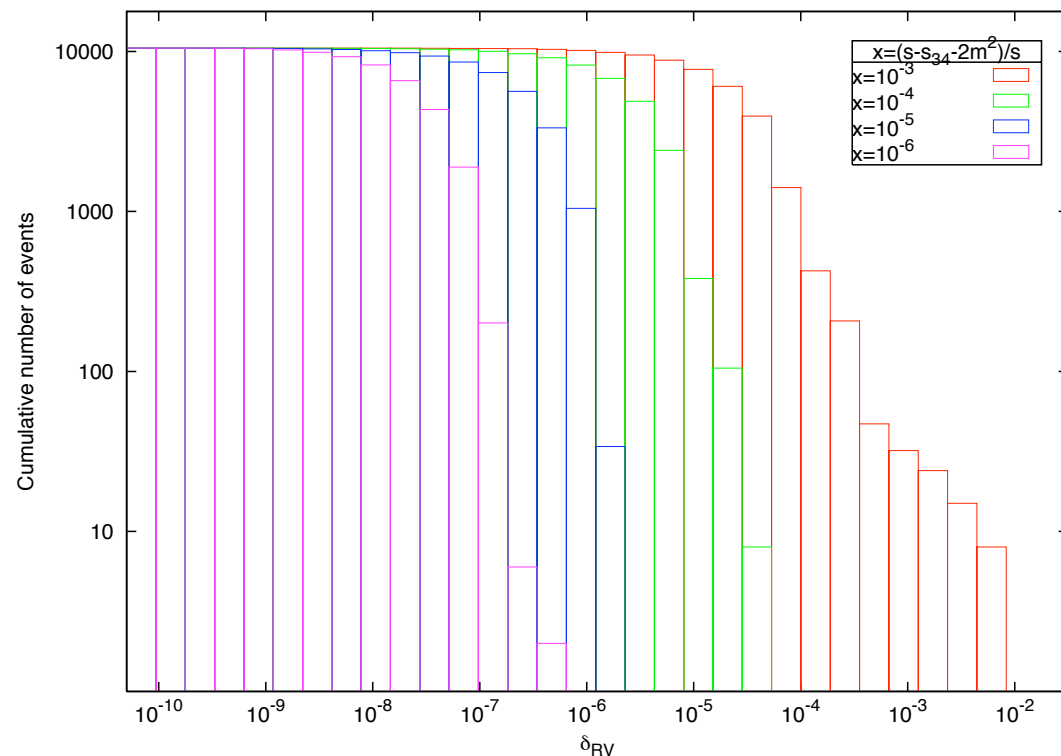
$$\mathcal{Poles} \left(d\hat{\sigma}_{NNLO}^{RV} - d\hat{\sigma}_{NNLO}^{VS} + d\hat{\sigma}_{NNLO}^{MF,1} + \int_1 d\hat{\sigma}_{NNLO}^{S,1} \right) = 0$$

Real Virtual Contributions

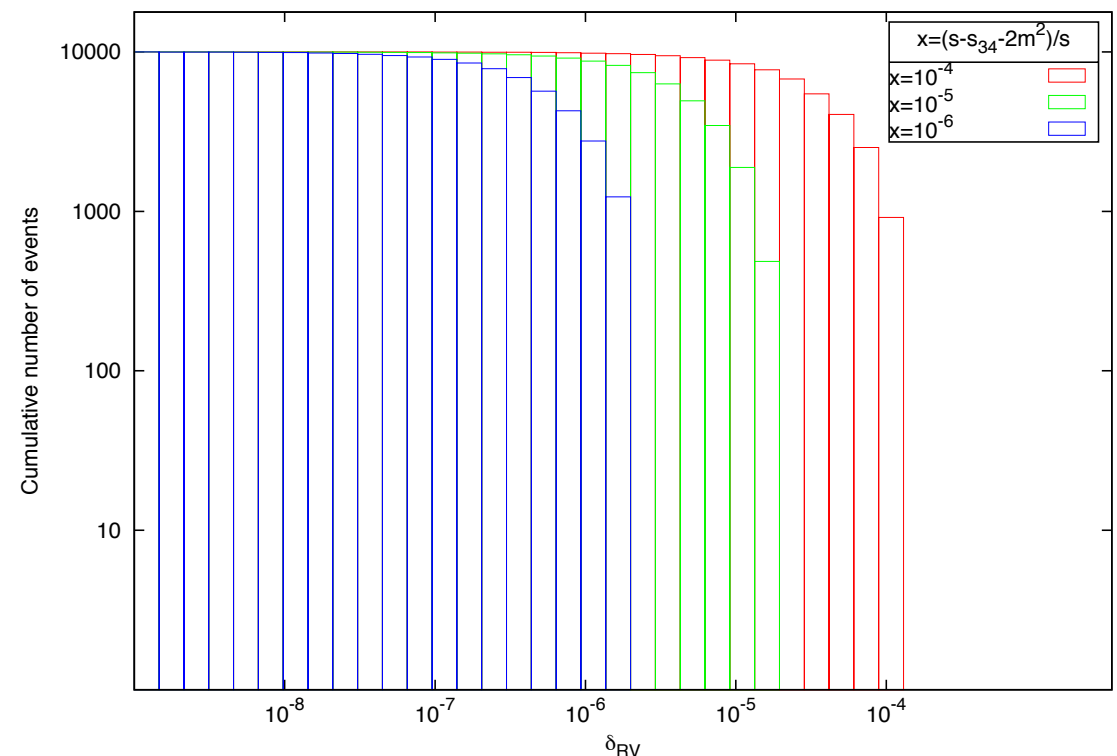
- Check of **convergence**. Analogous to double real check
 - ▶ **Good convergence observed in N_l piece** in soft and collinear limits
 - ▶ **Good convergence observed in collinear limit of leading color piece**
 - ▶ Convergence in **soft limit of leading-color piece** only achieved evaluating $d\hat{\sigma}_{NNLO}^{RV}$ in **quadruple precision**

$$\delta_{RV} = \left| \frac{\mathcal{F}inite(d\hat{\sigma}_{NNLO}^{RV})}{\mathcal{F}inite(d\hat{\sigma}_{NNLO}^T)} - 1 \right|$$

Soft limit (leading-color)



Soft limit (N_l)



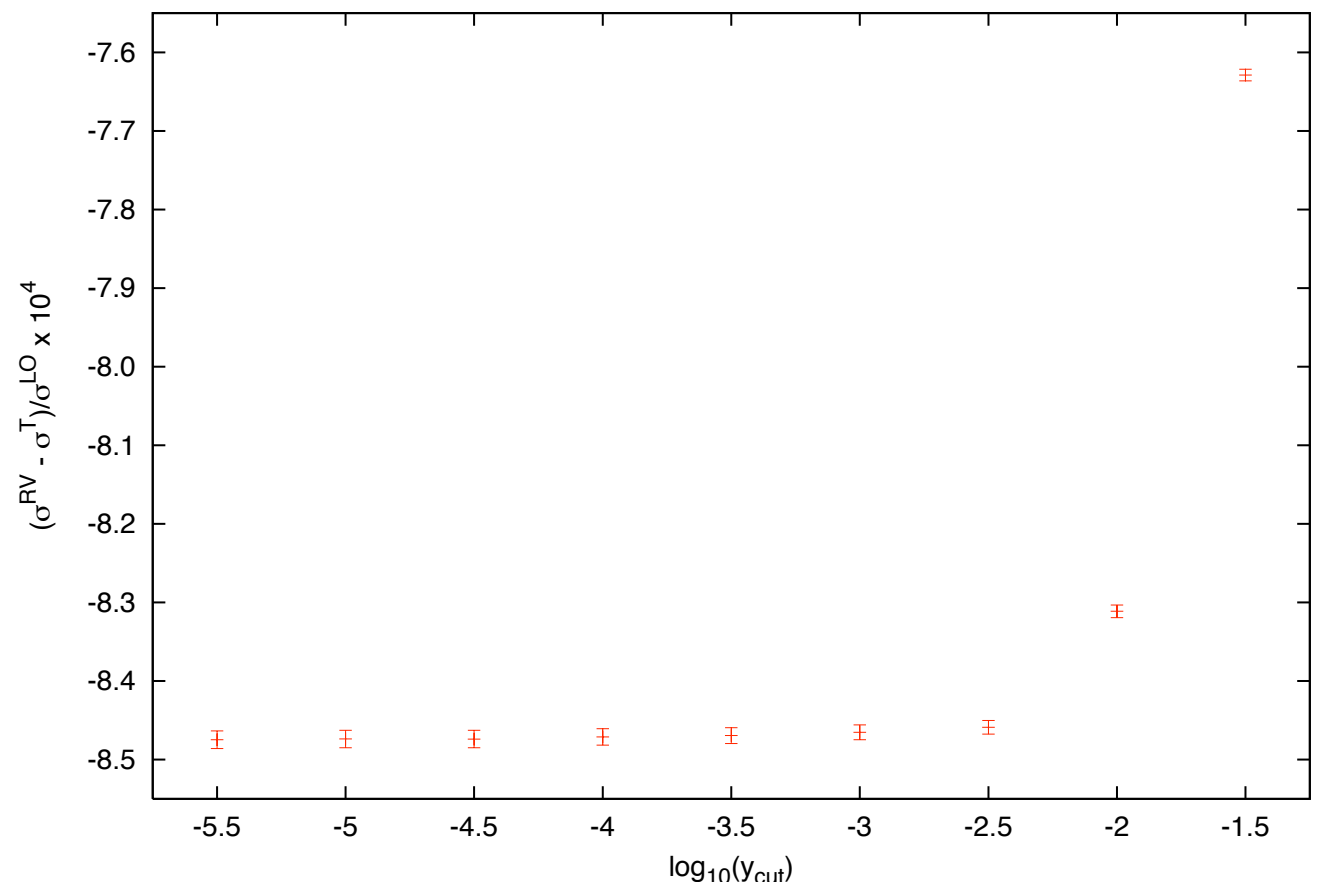
Real Virtual Contributions

Only “bad points” are (re)evaluated by OpenLoops in quadruple precision

- Fraction of quadruple precision evaluation in $\int_{d\Phi_3} \left(d\hat{\sigma}_{NNLO}^{RV} - d\hat{\sigma}_{NNLO}^T \right) ?$
- Is the integration stable?

Stability check: Evaluate $R = \left(\sigma_{NNLO}^{RV} - \sigma_{NNLO}^T \right) / \sigma_{LO}$ as a function of $y_{cut} = p_T^g / \sqrt{\hat{s}}$

- Integration is **stable**
- **R** has a plateau for $y_{cut} < y_{cut}^{max} \sim 10^{-3}$
- **Strong check** of our subtraction terms
- We can run with $y_{cut} \sim 10^{-4}$. **Only ~0.01% points require quadruple precision.**
- Efficient evaluation in **double precision** for the vast majority of points



Double Virtual Contributions

The ultimate check:

$$\mathcal{Poles} \left(d\hat{\sigma}_{NNLO}^{VV} + d\hat{\sigma}_{NNLO}^{MF,2} + \int_1 d\hat{\sigma}_{NNLO}^{VS} + \int_2 d\hat{\sigma}_{NNLO}^{S,2} \right) = 0$$

- Pole cancellation verified analytically in Nl piece

```
Pole1LC =  
Simplify[  
  Simplify[Coefficient[TwoLoopPolesLC, ep, -1] Delta[1 - x1] Delta[1 - x2] + Coefficient[OneTimesOneLoopPolesLC, ep, -1] Delta[1 - x1] Delta[1 - x2] -  
    Coefficient[SubTermLC, ep, -1]] /. ReplsLogsLC]  
0
```

- ▶ Non-trivial check on new integrated massive antennae
- ▶ Proves applicability of NNLO antenna subtraction to reactions with massive fermions
- Pole cancellation in leading color contributions in progress

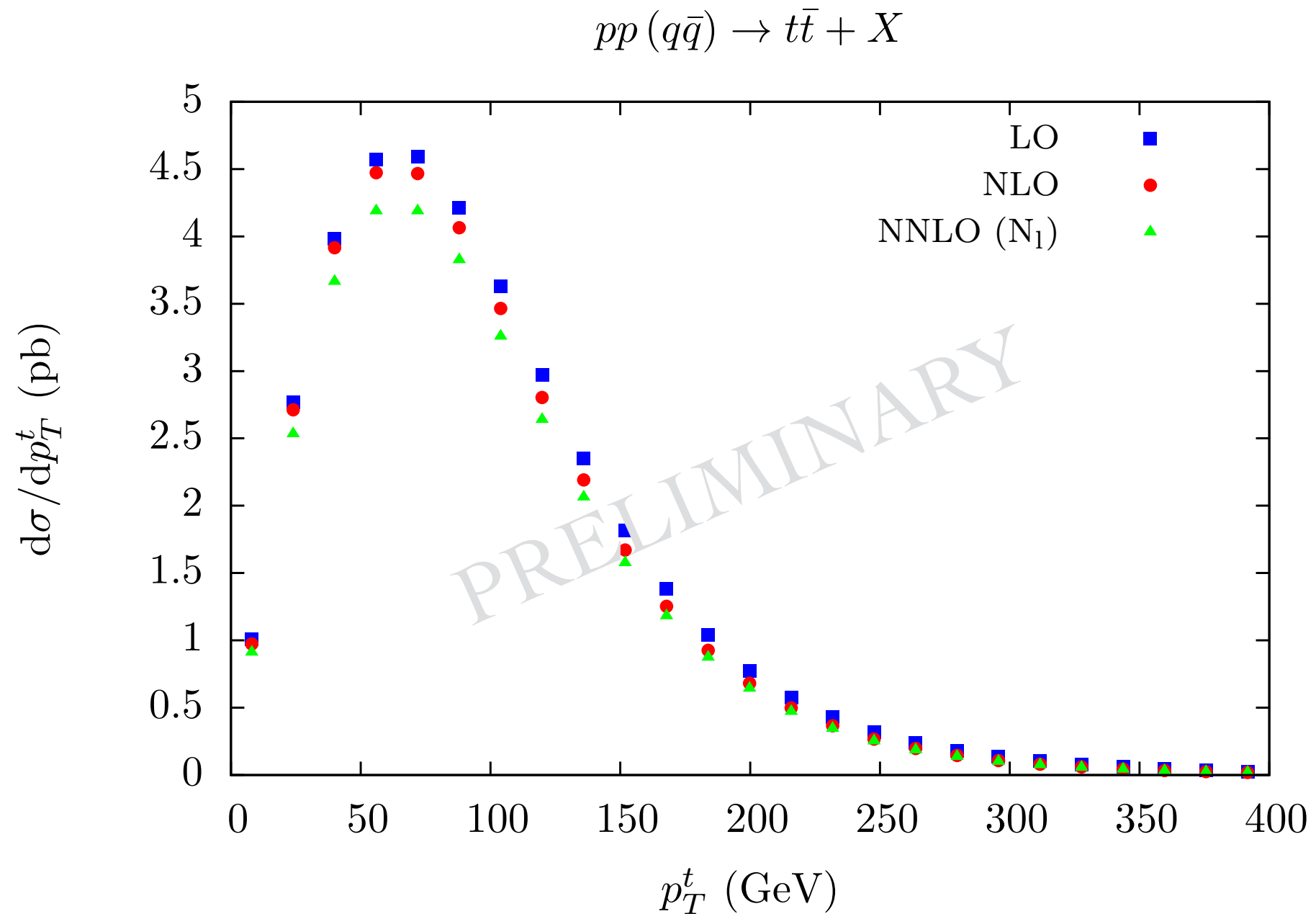
Preliminary Results

- Preliminary results for $pp(q\bar{q}) \rightarrow t\bar{t} + X$ (**N_l only**)
 - ▶ $\sqrt{s} = 8 \text{ TeV}$
 - ▶ $m_{top} = 173.5 \text{ GeV}$
 - ▶ $\mu = m_{top}$
 - ▶ MSTW2008NNLO PDF sets

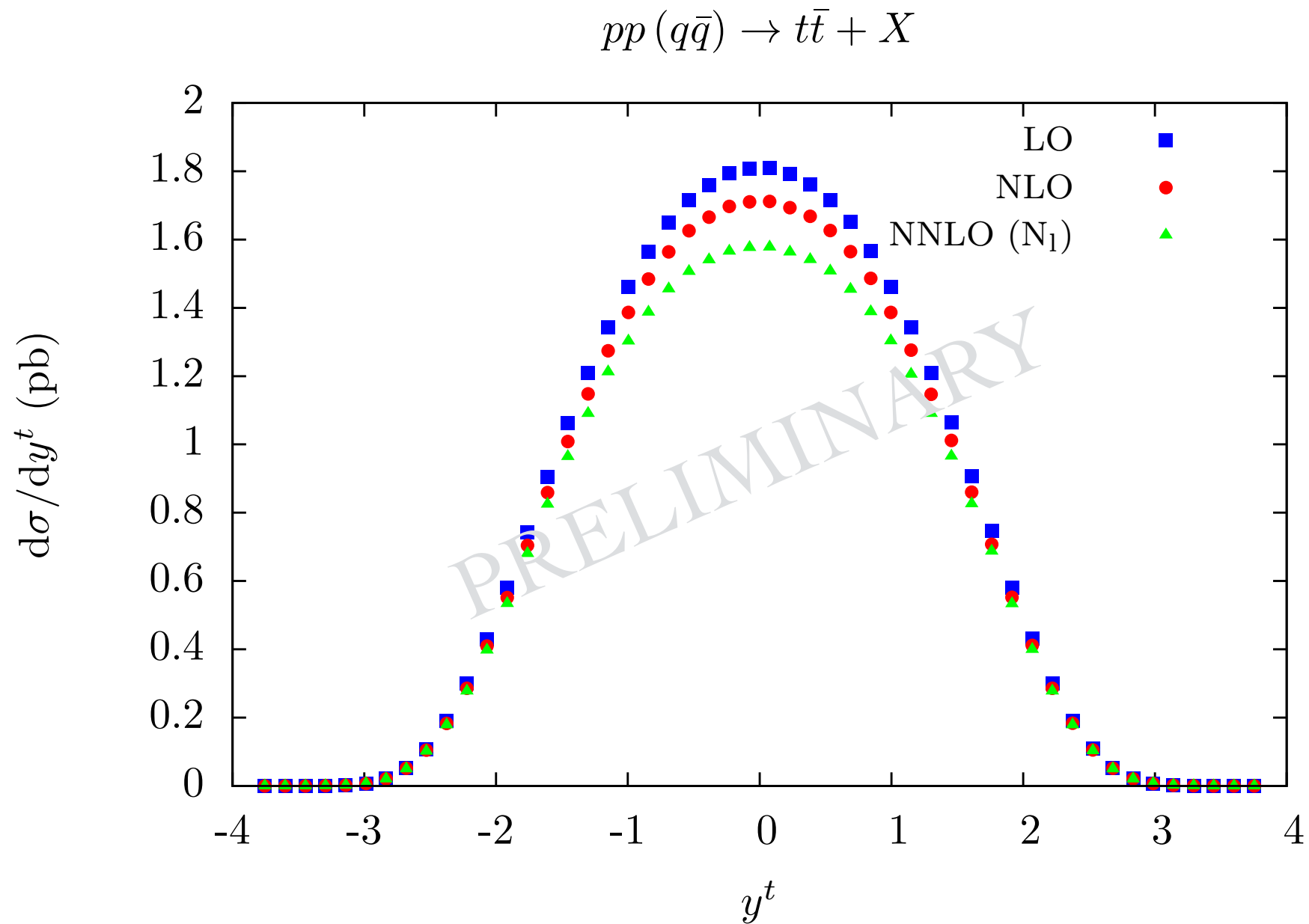


Warning: The following NNLO results contain the N_l contributions to the $q\bar{q}$ channel only. They are presented as a proof of principle. Strong phenomenological conclusions are not recommended.

Preliminary Results

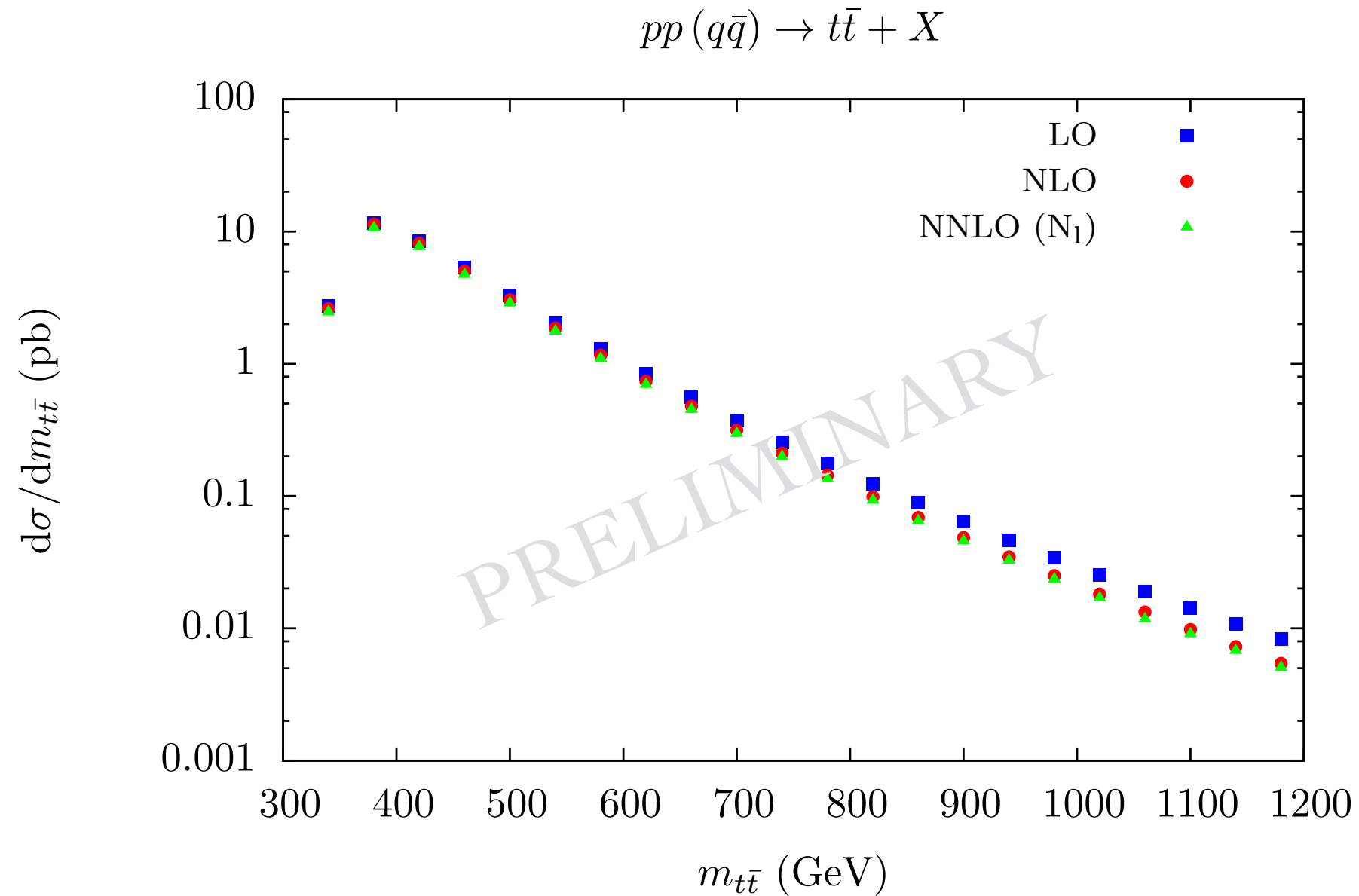


Preliminary Results



Could NNLO effects have an impact on our theory prediction for A_{FB} at Tevatron?

Preliminary Results



Summary And Outlook

- Fully differential NNLO calculation for $t\bar{t}$ production in the $q\bar{q}$ channel within reach (leading-color + fermionic contributions)
- Double real contributions: subtraction terms implemented and tested
- Real-virtual contributions:
 - ▶ Subtraction terms implemented and tested
 - ▶ Precise and stable one-loop amplitudes from OpenLoops in leading-color part
- Double virtual contributions:
 - ▶ Two-loop amplitudes available (for leading color and fermionic pieces)
 - ▶ Analytic pole cancelation in N_1 part
- Event generator implemented and working (with N_1 part for the moment)

Outlook

- Complete leading-color double virtual contributions in $q\bar{q}$ channel
- Phenomenology in $q\bar{q}$ channel. A_{FB} , main goal
- Include gg and qg channels